B.Sc. Semester-II Examination, 2023

MATHEMATICS [Honours]

Course ID : 22111 **Course Code : SH/MTH/201/C-3**

Course Title : Real Analysis

[OLD SYLLABUS]

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meaning.

UNIT-I

1. Answer any **five** from the following questions:

2×5=10

a) Show that the $\mathbb{N}$ (Set of Natural numbers) has no limit points.

b) Find the derived set of the set $S = \{\frac{1}{3^n} + \frac{1}{5^m} : m, n \in \mathbb{N}\}$.

c) Give an example to show that arbitrary union of compact sets may not be compact.

d) Test the convergence of the series $\frac{1}{1.2.3} + \frac{2}{2.3.4} + \frac{3}{3.4.5} + \ldots\ldots$

e) Find $\overline{lim}$ and $\underline{lim}$ of the sequence $\{(-1)^n (1 + \frac{1}{n})\}$.

f) Test the convergence of the series $1 + \frac{1}{1!} + \frac{2^2}{2!} + \frac{3^3}{3!} + \cdots$.

g) Prove that if x is an arbitrary real number, there is a sequence $\{r_n\}$ of rational numbers converging to x.

h) Test the convergence of the series $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \cdots$.

UNIT-II

2. Answer any **four** from the following questions:

5×4=20

a) Show that the sequence $\left\{\left(1 + \frac{1}{n}\right)^n\right\}$ is convergent.

b) Prove that the set (0, 1) is uncountable.

c) Show that $\{u_n\}$, where $u_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots . + \frac{1}{n!}$, is a Cauchy sequence.

d) i) Prove that $\lim\limits_{n\to\infty} \frac{(n!)^{\left(\frac{1}{n}\right)}}{n} = \frac{1}{e}$

ii) Show that every infinite bounded subset of R has at least one limit point in R. 2+3

e) Prove that $\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \cdots$, using the expansion of $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$.

f) Prove that intersection of a finite collection of closed sets is a closed set, but intersection of an infinite collection of closed sets may not be closed always.

g) Using Cauchy's integral test, discuss the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ for different values of p.

UNIT-III

3. Answer any **one** of the following questions:

10×1=10

a) i) If S be a bounded subset of R, then show that derived set S′ of S, is also an abounded set.

ii) Determine the derived set of the set $S = \left\{\frac{1}{m} + \frac{1}{n} : m, n \in N\right\}$.

iii) Test the convergence of the series $1 + \frac{x}{1!} + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!} + \cdots$, for different values of *x*.

3+2+5=10

b) i) Prove that a real number c is a limit point of a set A iff every neighbourhood of c contains infinitely many points of A.

ii) If $a_1 = \sqrt{3}$ and $a_{n+1} = \sqrt{3}(a_n)$ then show that $\lim_{n\to\infty} a_n = 3$.

iii) Test the convergence of the series $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \ldots$ *for* $0 < x \le 1$.

4+3+3

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